AE2403 VIBRATIONS AND ELEMENTS OF AEROELASTICITY L T P C
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OBJECTIVE

To study the dynamic behaviour of different aircraft components and the interaction among the aerodynamic, elastic and inertia forces

UNIT I BASIC NOTIONS 8

Simple harmonic motion – Terminologies – Newton's Law – D' Alembert's principle – Energy Methods

UNIT II SINGLE DEGREE OF FREEDOM SYSTEMS 12

Free vibrations – Damped vibrations – Forced Vibrations, with and without damping – support excitation – Vibration measuring instruments.

UNIT III MULTI DEGREES OF FREEDOM SYSTEMS 10

Two degrees of freedom systems – Static and Dynamic couplings vibration absorber-Principal co- ordinates, Principal modes and orthogonal condition – Eigen value problems.

Hamilton's principle- Lagrangean equation and application – Vibration of elastic bodies-Vibration of strings- Longitudinal, Lateral and Torsional vibrations.

UNIT IV APPROXIMATE METHODS 5

Rayleigh's and Holzer Methods to find natural frequencies.

UNIT V ELEMENTS OF AEROELASTICITY 10

Concepts – Coupling – Aero elastic instabilities and their prevention – Basic ideas on wing divergence, loss and reversal of aileron control – Flutter and its prevention.

TOTAL: 45 PERIODS

TEXT BOOKS

1. Timoshenko S., "Vibration Problems in Engineering"– John Wiley and Sons, New York, 1993.
2. Fung Y.C., "An Introduction to the Theory of Aeroelasticity" – John Wiley & Sons, New York, 1995.

REFERENCES

1. Bisplinghoff R.L., Ashley H and Hoffman R.L., "Aeroelasticity" – Addision Wesley Publication, New York, 1983.
2. Tse. F.S., Morse, I.F., Hinkle, R.T., "Mechanical Vibrations", – Prentice Hall, New York, 1984.
3. Scanlan R.H. & Rosenbaum R., "Introduction to the study of Aircraft Vibration & Flutter", John Wiley and Sons. New York, 1982.
4. Tongue. B. H., "Principles of Vibration", Oxford University Press, 2000.

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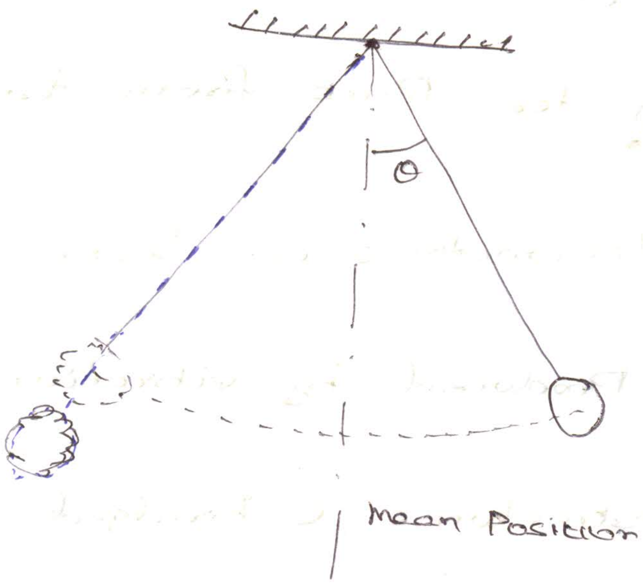
VIBRATION AND AEROELASTICITY

VIBRATION:

The motion which repeats itself after an interval of time is called vibration or oscillation

Eg: When body displaced by external force, the internal forces in form of elastic energy tends to bring the body original position. At equilibrium position the whole of elastic energy converted into kinetic energy and the body continues to move in the opposite direction



SIMPLE PENDULUM

Reasons for vibration:

1. Unbalanced centrifugal force on the system. This is due to non-uniform material distribution in a rotating machine element.

2. Elastic nature of the system

3. External excitation applied on the system

4. Winds may causes vibration such as electric line, telephone line etc

Causes of vibration:

1. Rapid wear of machine parts such as bearing and gears

2. Loosening the parts from the machine

3. Wheels of locomotives can leave the track

4. Noise produced by vibration

5. Many structures & bridged falls because of vibration.

USEFUL PURPOSE

Vibration testing equipments

Vibratory conveyors,

hoppers, sieves and

compactors.

Musical instruments

Geological research (earthquakes)

Elimination of vibration:

1. Removing external excitation, if possible

2. Using shock absorbers

3. Dynamic absorbers

4. Resting the system on proper vibration isolator.

Frequency:

The number of cycle that a vibrating body completes in one second is called frequency. (Hz) Hertz

One Hertz = one cycle/second

How frequent that event occurs

Terminologies

Periodic motion:

A motion which repeats itself after equal interval of time

Time Period:

Time taken to complete one cycle

Amplitude!

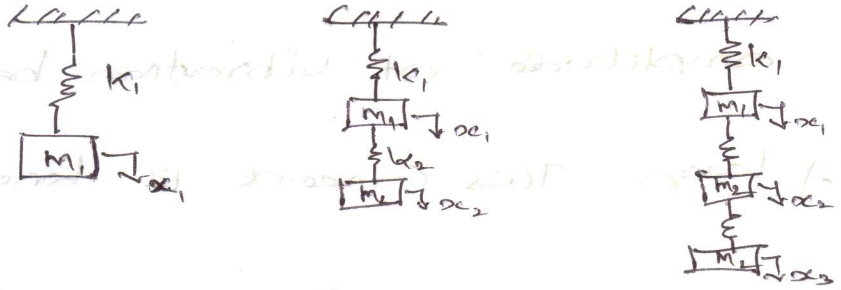
The maximum displacement of a vibrating body from its equilibrium position

Natural frequency!

When no external force act on the system after giving it an initial displacement the body vibrates. These vibration is called Free vibration and their frequency as natural frequency. It expressed in rad/sec (or) Hertz.

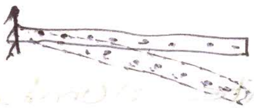
Degree of freedom!

The minimum number of independent coordinates required to specify the motion of a system at any instant is known as degree of freedom of the system.



One　　　Two　　　Three (degree of freedom)

x_1, x_2 & x_3 independent coordinate.



Cantilever beam having infinite degree of freedom.

Damping:

It is the resistance to the motion of a vibrating body.

The vibration associated with the resistance are known as damped vibration.

Phase difference:

The two vectors x_1 and x_2 having frequencies ω rad/sec. The vibration motion can be expressed as

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin(\omega t + \phi)$$

ϕ - Phase difference

Resonance:

When the frequency of external excitation is equal to natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This concept is know as Resonance.

Continous & Discrete System

The mechanical system includes elastic members which have infinite number of degree of freedom and is called continous

Eg: cantilever, simply supported beam.

Mechanical System:

The system consisting of mass, stiffness and damping are known as mechanical system

Discrete or lumped system:

System with finite number of degree of freedom are called discrete or lumped system.

Simple Harmonic motion:

The motion of a body to and fro about a fixed Point is called simple harmonic motion. The motion is Periodic and its acceleration is always directed towards the mean Position and is Proportional to its distance from mean Position.

$$x = A \sin \omega t$$

$$\dot{x} = A \omega \cos \omega t$$

$$\ddot{x} = -A \omega^2 \sin \omega t$$

$$\ddot{x} = -\omega^2 x$$

x = displacement =

$\dot{x}$ = Velocity $= \frac{dx}{dt}$

$\ddot{x}$ = acceleration $\frac{d^2x}{dt^2}$

1. Simple Harmonic motion is a Special Case of Periodic motion
2. Simple harmonic motion requires a restoring force, but there can be Periodic motion without restoring force

(Spring, gravitational force, magnetic, electric

Types of Vibration

Free or Forced vibration

Linear and non-Linear vibration

Damped and undamped vibration

Deterministic & Random vibration

Longitudinal, Transverse & Torsional Vibration

Free vibration:

After disturbing the system the external excitation is removed, then the system vibrate on its own is called free vibration.

Forced vibration:

The vibration which is under the influency of external force is called forced vibration

Eg: machine tools, electric bells.

Linear vibration:

If in a vibratory system mass, spring and damper behave in a linear manner, the

Vibration caused are known as linear in nature

Non linear vibration:

If any basic components of a vibratory system behaves non-linearly, the vibration is called non-linear.

Note: Linear vibration becomes non-linear for very large amplitude of vibration. It does not follow the law of superposition

Damped:

If the vibratory system has a damper, the motion of the system will be opposed by it and the energy of the system will be dissipated in friction, and is called damped vibration.

No damper is known as undamped vibration.

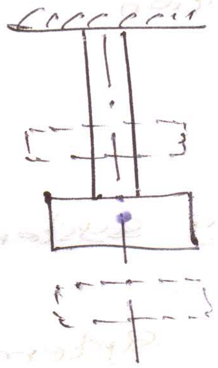
Deterministic:

If in the system the amount of external excitation is known in magnitude, it causes deterministic vibration.

The non-deterministic vibration are known as random vibration.

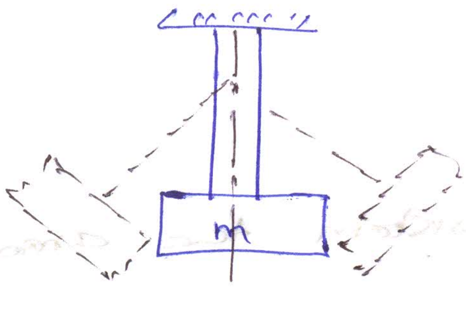
Longitudinal:

If a mass moves up & down parallel to the spindle axis, it said to be longitudinal



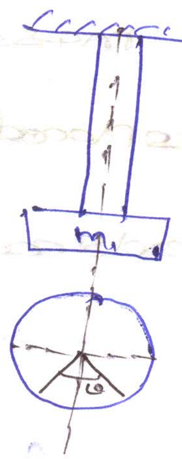
Transverse:

When the particle of the body or shaft moves approximately perpendicular to the axis of the shaft.



Torsional vibration!

If the spindle gets alternately twisted and untwisted on account of vibratory motion of the suspended disc.

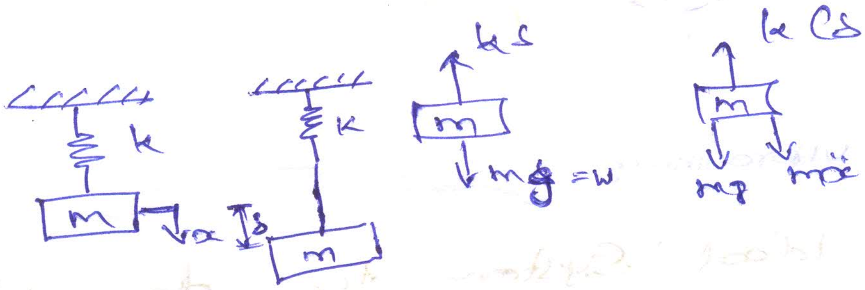


Transient vibration:

In ideal system the free vibrations continue indefinitely as there is no damping. The amplitude of vibration decays continously because of damping and vanish ultimately. Such vibration in real system is called transient vibration

NEWTON'S METHOD:

Consider the spring mass system to move in a rectilinear manner, along the axis of the spring. The spring of constant stiffness "k" which is fixed at one end and carries mass at another end. The body is displaced from its equilibrium position vertically. This is said to be static



$mg = k\delta = W$

$W = k\delta$

W = gravitation pos

$$m\ddot{x} = mg - k(\delta + x)$$

$$= k\delta - k\delta - kx$$

$$m\ddot{x} = -kx$$

$$m\ddot{x} + kx = 0 \quad \text{—} \ (1)$$

$$x = A\cos\omega_n t + B\sin\omega_n t$$

$$x = \frac{dx}{dt} \quad \dot{x} = -\omega_n A \sin\omega_n t + \omega_n B\cos\omega_n t \quad \text{—} \ (2)$$

$$\ddot{x} = \frac{d^2x}{dt^2}$$

$$\ddot{x} = -\omega^2_n (A \cos \omega_n t + B \sin \omega_n t) \quad \text{--- (3)}$$

Sub (2) & (3) in (1)

$$-m(\omega^2_n (A \cos \omega_n t + B \sin \omega_n t) + k(A \cos \omega_n t + B \sin \omega_n t) = 0$$

$$-\omega^2_n (A \cos \omega_n t + B \sin \omega_n t) + k/m (A \cos \omega_n t + B \sin \omega_n t) = 0$$

$$-\omega^2_n + k/m = 0$$

$$\boxed{\omega_n = \sqrt{k/m}} \qquad \boxed{F = 1/2\pi \; \omega_n}$$

$$\boxed{F = 1/2\pi \sqrt{k/m}}$$

x = Displacement, δ = Static deflection of the spring

$m\ddot{x}$ = Restoring force

Energy method.

Assume the system to be a conservative one. In a conservative system the total sum of the energy is constant.

The vibratory system the energy is partly potential and partly kinetic.

The kinetic energy T is because of velocity of the mass

Potential energy V is stored in the spring because of its elastic deformation.

Conservation law of energy, Accordingly,

$$T + V = \text{Const}$$

Differentiating w.r.t time

$$\frac{d}{dt}(T + V) = 0$$

$$T = \tfrac{1}{2} m \dot{x}^2$$

$$V = \tfrac{1}{2} k x^2$$

$$\frac{d}{dt}\left(\tfrac{1}{2} m \dot{x}^2 + \tfrac{1}{2} k x^2\right) = 0$$

$$m \ddot{x} \dot{x} + k x \dot{x} = 0$$

$$\boxed{m \ddot{x} + k x = 0}$$

Equation of motion

$$x = A \sin \omega t$$

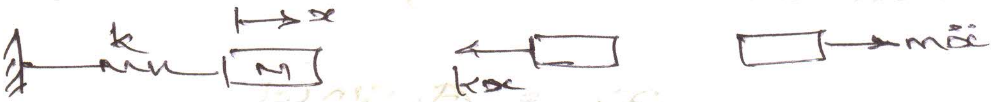
$$\ddot{x} = -A \omega^2 \sin \omega t$$

$$-m A \omega^2 \sin \omega t + k A \sin \omega t = 0$$

$$\boxed{\omega = \sqrt{k/m} \text{ rad/s}} \quad \boxed{f = \frac{1}{2\pi}\sqrt{k/m} \text{ Hz}}$$

Spring-mass System in Horizontal Position

The body of mass "m" is free to move on a fixed horizontal surface. The mass is supported on frictionless roller. The spring of constant stiffness "k" is attached to a fixed frame at one side and mass at other side.



$$m\ddot{x} = -kx$$

$$m\ddot{x} + kx = 0$$

It shows that frequency is same for horizontal & vertical position.

Periodic & Harmonic motion:

The motion which repeats itself after an equal interval of time is known as Periodic motion.

The equal interval of time is called time Period. If a motion, of the type

$$x_1 = A_1 \sin \omega t$$

x_1 = is the displacement

A_1 = Amplitude

$$\dot{x}_1 = \frac{dx_1}{dt} = A_1 \omega \cos \omega t \quad \text{[velocity]}$$

$$\ddot{x} = \frac{d^2 x_1}{dt^2} = -\omega^2 A_1 \sin \omega t$$

$$= -\omega^2 x_1 \quad \text{[Acceleration]}$$

All harmonic motion are Periodic in nature but vice-versa is not always ~~[illegible]~~ true.

The acceleration in a simple harmonic motion is proportional to its displacement and directed towards a particular fixed point.

Problem:

Add the following harmonic motions analytically and check the solution graphically

$$x_1 = 4\cos(\omega t + 10^\circ)$$

$$x_2 = 6\sin(\omega t + 60^\circ)$$

Solution:

The frequency is same for both x_1 & x_2

So we express the sum as

$$x = A\sin(\omega t + \alpha)$$

$$x = x_1 + x_2$$

$$A(\sin\omega t\cos\alpha + \cos\omega t\sin\alpha) = 4\cos(\omega t + 10^\circ) + 6\sin(\omega t + 60^\circ)$$

$$= 4\cos\omega t\cos 10^\circ - 4\sin\omega t\sin 10^\circ + 6\sin\omega t\cos 60^\circ + 6\cos\omega t\sin 60^\circ$$

$$= \sin\omega t(-4\sin 10^\circ + 6\cos 60^\circ) + \cos\omega t(4\cos 10^\circ + 6\sin 60^\circ)$$

$$= \sin\omega t(-0.6945 + 3) + \cos\omega t(3.9392 + 5.19)$$

$$= \sin\omega t(2.305) + \cos\omega t(9.135)$$

Equating corresponding coefficient of $\cos\omega t$ & $\sin\omega t$

$$A\cos\alpha = 2.305$$

$$A\sin\alpha = 9.135$$

$$A = \sqrt{(2.305)^2 + (9.135)^2}$$

$$= \sqrt{88.7612} = 9.42$$

$$\tan\alpha = \frac{9.135}{2.305} = 3.963$$

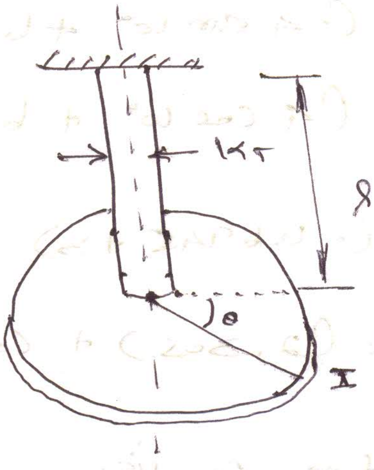
$$\alpha = \tan^{-1}(3.963)$$

$$\alpha = 75.838^\circ$$

$$x = 9.42\sin(\omega t + 75.838^\circ)$$

TORSIONAL VIBRATION

The rotor of mass moment of inertia 'I' connected to a shaft (at its end) of torsional stiffness k_T is twisted by an angle θ as shown in fig



$$I\ddot{\theta} = -k_T\theta$$

$$I\ddot{\theta} + k_T\theta = 0$$

$$\ddot{\theta} + \frac{k_T}{I}\theta = 0 \quad \text{—— (1)}$$

$$\omega_n^2 = \frac{k_T}{I}$$

Equation written (1)

$$\ddot{\theta} + \omega_n^2\theta = 0$$

$$\omega_n = \sqrt{\frac{k_T}{I}}$$

$$\frac{T}{J} = \frac{G\theta}{I} \quad \text{or} \quad k_T = \frac{T}{\theta} = \frac{GJ}{l}$$

$$k_T = \frac{GJ}{l} = \frac{G}{l}\left(\frac{\pi}{32}d^4\right)$$

T - Torque , J - Polar moment of merit

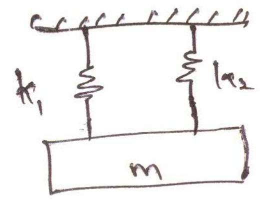
θ - angle of Shaft , l - length of the Shaft

G - modulus of rigidity

EQUIVALENT STIFFNESS OF SPRING COMBINATIONS

Some system have more than one spring. The spring are joined in series or parallel or both. They can be replaced by a single spring of same stiffness.

Springs in Parallel

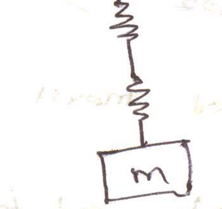


The deflection of individual spring is equal to the deflection of the system.

$$k_1 x + k_2 x = k_e x$$

$$k_e = k_1 + k_2$$

Springs in Series:



$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

When springs connected in series the reciprocal of equivalent spring stiffness is equal to the sum of the reciprocals of individual spring stiffnesses.

k_e = equivalent stiffness of the system

k_1, k_2 = stiffness of springs

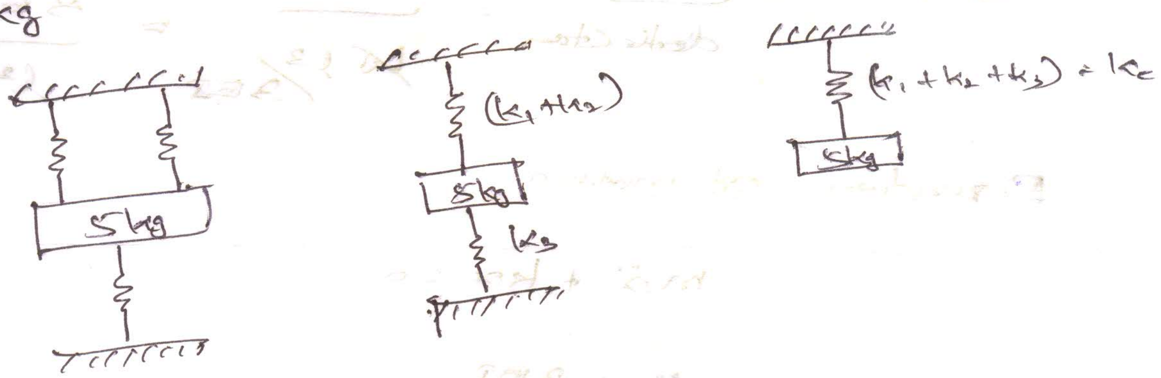
x = deflection of the system

$k_e x$ = Force on the system.

Find the natural frequency of the system

Shown in fig: $k_1 = k_2 = 1500\ N/m$, $k_3 = 2000\ N/m$

$m = 5\ kg$



$m = 5\ kg$

Equivalent stiffness in Parallel

$k_e = k_1 + k_2 + k_3$

$= 1500 + 1500 + 2000$

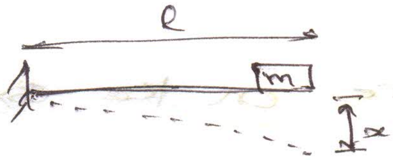
$= 5000\ N/m$

$$\frac{N/m}{kg} = \frac{kg\ m/s^2 / m}{kg}$$

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{5000}{5}} = 31.62\ rad/sec$$

$$f_n = \frac{1}{2\pi}\omega_n = 5.03\ Hz$$

Determine the natural frequency of the mass 'm' placed at one end of a cantilever beam of negligible mass as shown in fig



Deflection $\delta = \dfrac{W l^3}{3EI}$

EI flexural rigidity

$$\text{Stiffness} = \frac{\text{load}}{\text{deflection}} = \frac{W}{W l^3 / 3EI} = \frac{3EI}{l^3} = k$$

Equation of motion

$$m\ddot{x} + kx = 0$$

$$m\ddot{x} + \frac{3EI}{l^3}x = 0$$

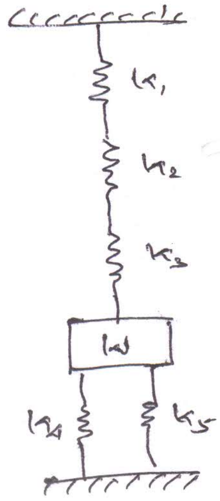
$$\omega_n = \sqrt{\frac{3EI}{l^3 m}} \text{ rad/sec}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{m l^3}} \text{ Hz}$$

1. A Pendulum is observed to complete 23 full cycle in 58 sec. Determine the period and the frequency of the Pendulum

   Frequen = $23/58$ = 0.40 Hz ; Period = $58/23$ = 2.5 s

2. A mass is tied to a spring mass begins vibrating Periodically. The distance b/w its highest and the lowest Position in 38 cm. What is the amplitude of vibration? Ans = 19 cm



$k_1 = 2$ kgf/cm, $k_2 = 1.5$ kgf/cm

$k_3 = 3.0$ kgf/cm, $k_4 = k_5 = .5$ kgf/cm

$f_n = 10$ Hz

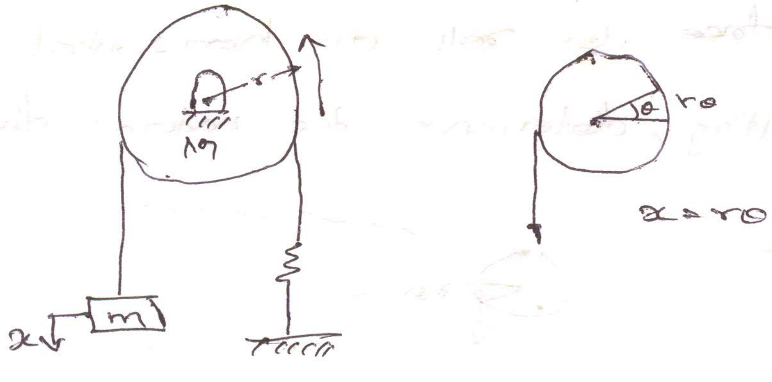
Solution: $K_{e_1} = 0.667$ kgf/cm, $K_{e_2} = 1.0$ kgf/cm

$1/K_{e_1} = 1.499$

$$(\omega^2) = \frac{1}{4\pi^2}\,\frac{1.667 \times 9.80 \times 100}{\cancel{100 \times 4\pi^2}\; W}$$

$$W = \frac{1.667 \times 9.80 \times 100}{100 \times 4\pi^2} = 0.414 \text{ kg}$$

Determine the natural frequency of the spring mass Pulley System shown in fig



T = K.E of mass m + K.E of Pulley M + P.E of Spring k

$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k x^2$

$= \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k r^2 \theta^2$

Moment of inertia of Pulley $I = \frac{1}{2} M r^2$

$$\frac{d}{dt}\left[\frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k r^2 \theta^2\right] = 0$$

$$m r^2 \ddot{\theta}\dot{\theta} + I \ddot{\theta}\dot{\theta} + k r^2 \dot{\theta}\theta = 0$$

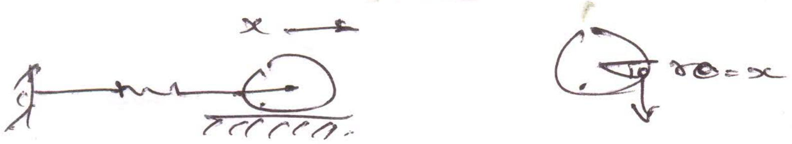
$$m r^2 \ddot{\theta} + I \ddot{\theta} + k r^2 \theta = 0$$

$$\left(m r^2 + \frac{1}{2} M r^2\right)\ddot{\theta} + k r^2 \theta = 0$$

$$\ddot{\theta} + \frac{k r^2}{m r^2 + \frac{1}{2} M r^2}\theta = 0$$

$$\omega_n = \sqrt{\frac{2 k r^2}{2 m r^2 + M r^2}} = \sqrt{\frac{2k}{2m + M}} \text{ rad/sec}$$

Q. A circular cylinder of mass 4 kg and radius 15 cm is connected by a spring of stiffness 4000 N/m as shown in fig. It is free to roll on horizontal rough surface without slipping, determine the natural frequency.



$$\omega_n = \sqrt{2k/3m} \text{ rad/s}$$

Damping:

Damping is the resistance offered by a body to the motion of a vibratory system. The resistance may be applied by a ~~gas~~ liquid or solid internally or externally.

The main advantage of providing damping in mechanical system is just to control the amplitude of vibration.

Types of Damping:

Viscous Damping

Coulomb Damping

Structural Damping

Non linear, slip or interfacial damping

Viscous Damping:

When the system is allowed to vibrate in a viscous medium, the damping is called a viscous. Viscosity is the property of a fluid by virtue of which it offers resistance to the motion of one layer over the adjacent one.

Coulomb Damping

When one body is allowed to slide over the other, the surface of one body offer some resistance to the movement of the other body on it. This resistance force is called force of friction.

Sometimes it also known as dry friction.

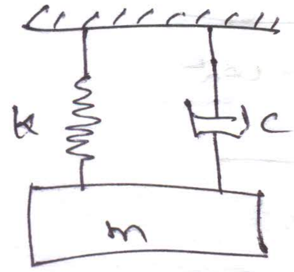
Structural Damping:

The resistance is offered by the elastic properties from within the body. There is intermolecular friction in the structure which oppose the movement. The magnitude of damping is very small as compared to other damping.

Non-linear, Slip or Interfacial Damping

The machine elements are connected through various types of joints. microscopic slip occurs on the interfaces of machine elements which causes dissipation of vibration energy.

Free damped vibration



C - Damping co-efficient N sec/m

k : Stiffness N/m

m : mass N s²/m

The equation of motion is given as

$$m\ddot{x} + c\dot{x} + kx = 0$$

$\div m$
$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \omega_n^2 x = 0 \qquad \xi = \text{zetta}$$

take

$$2\xi\omega_n = \frac{c}{m}$$

$$\therefore \ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0$$

ξ is known as damping factor which is non-dimensional

now, Put $x = e^{st}$; $\dot{x} = se^{st}$; $\ddot{x} = s^2e^{st}$

$$s^2\cancel{e^{st}} + 2\xi\omega_n s\cancel{e^{st}} + \omega_n^2\cancel{e^{st}} = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$S = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\xi\omega_n \pm \sqrt{\xi^2\omega_n^2 - \omega_n^2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$\therefore S_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$

$$S_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$

Case 1 :- When $\xi > 1$

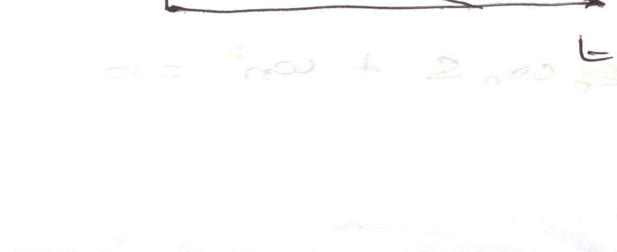
the roots are real and distinct

$$x = Ae^{S_1 t} + Be^{S_1 t}$$

$$= Ae^{(-\xi\omega_n + \omega_n\sqrt{\xi^2-1})t} + Be^{(-\xi\omega_n - \omega_n\sqrt{\xi^2-1})t}$$

$$= e^{-\xi\omega_n t}\left[Ae^{(\omega_n\sqrt{\xi^2-1})t} + Be^{(-\omega_n\sqrt{\xi^2-1})t}\right]$$

$x(t)$ decreases exponentially



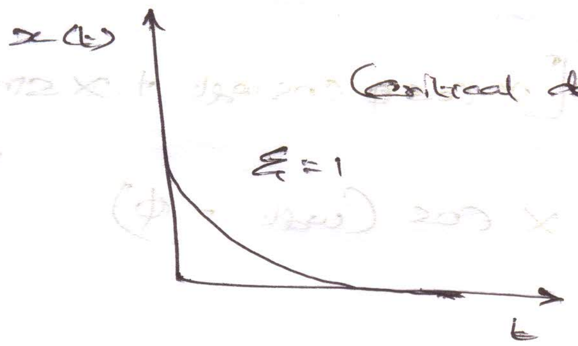
(over damping system)

Case 2 :-

$\xi = 1$, the roots are real & equal

$$x = e^{-\xi \omega_n t}(At + B)$$

The decay is much faster when compared to $\xi > 1$



Case 3 :-

$\xi < 1$, the roots are complex

$$S_1 = -\xi \omega_n + i\sqrt{1-\xi^2}\,\omega_n$$

$$S_2 = -\xi \omega_n - i\sqrt{1-\xi^2}\,\omega_n$$

$$x(t) = A e^{(-\xi\omega_n + i\sqrt{1-\xi^2}\,\omega_n)t} + B e^{(-\xi\omega_n - i\sqrt{1-\xi^2}\,\omega_n)t}$$

$$= e^{-\xi\omega_n t}\left[A e^{(+i\sqrt{1-\xi^2}\,\omega_n)t} + B e^{(-i\sqrt{1-\xi^2}\,\omega_n)t}\right]$$

$$= e^{-\xi\omega_n t}\left[A e^{i\theta} + B e^{-i\theta}\right] \quad \text{where } \theta = \sqrt{1-\xi^2}\,\omega_n$$

$$= e^{-\xi\omega_n t}\left[A\{\cos\omega_n\sqrt{1-\xi^2}\,t + i\sin\omega_n\sqrt{1-\xi^2}\,t\} + B\{\cos\omega_n\sqrt{1-\xi^2}\,t - i\sin\sqrt{1-\xi^2}\,t\}\right]$$

$\{e^{-i\theta} = \cos\theta - i\sin\theta\}$

$$= e^{-\xi\omega_n t}\left[(A+B)\cos\omega_n\sqrt{1-\xi^2}\,t + (A-iB)i\sin\omega_n\sqrt{1-\xi^2}\,t\right]$$

Taking $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$= e^{-\xi\omega_n t}\left[(A+B)\cos\omega_d t + (A-iB)i\sin\omega_d t\right]$$

Take

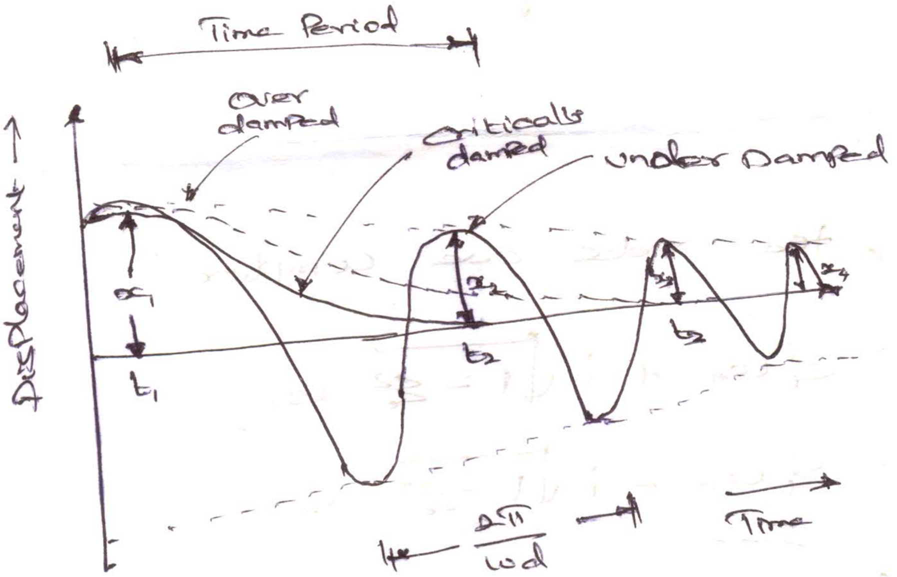
$$A+B = X\cos\phi$$

$$(A-iB)i = X\sin\phi$$

$$x(t) = e^{-\xi\omega_n t}\left[X\cos\phi\cos\omega_d t + X\sin\phi\sin\omega_d t\right]$$

$\boxed{\cos A-B \text{ formula}}$

$$x(t) = e^{-\xi\omega_n t}X\cos(\omega_d t-\phi)$$



$$x(t) = e^{-\xi\omega_n t}X\cos(\omega_d t-\phi)$$

ω_d = damped natural frequency

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

Let x_1 be the displacement at t_1

$$X_1 = X e^{-\xi\omega_n t_1}\cos(\omega_d t_1-\phi)$$

$$x_2 = X e^{-\xi \omega_n t_2} \cos(\omega_d t_2 - \phi)$$

Let x_2 be the displacement at t_2

$$x_2 = X e^{-\xi \omega_n \left(t_1 + \frac{2\pi}{\omega_d}\right)} \cos\left\{\omega_d\left(t_1 + \frac{2\pi}{\omega_d}\right) - \phi\right\}$$

$$= X e^{-\xi \omega_n \left(t_1 + \frac{2\pi}{\omega_d}\right)} \left(\cos\omega_d t_1 + \cos 2\pi - \cos\phi\right)$$

$$= X e^{-\xi \omega_n \left(t_1 + \frac{2\pi}{\omega_d}\right)} \cos\omega_d t_1 + 1 - \cos\phi$$

$$= X e^{\xi \omega_n \left(t_1 + \frac{2\pi}{\omega_d}\right)} \cos(\omega_d t_1 - \phi) + 1$$

$$\frac{x_1}{x_2} = \frac{\cancel{X} e^{-\xi \omega_n t_1} \cancel{\cos(\omega_d t_1 - \phi)}}{\cancel{X} e^{-\xi \omega_n \left(t_1 + \frac{2\pi}{\omega_d}\right)} \cancel{\cos(\omega_d t_1 - \phi)}}$$

$$= e^{-\xi \omega_n t_1} \cdot e^{\xi \omega_n \left(t_1 + \frac{2\pi}{\omega_d}\right)}$$

$$= \cancel{e^{\xi\left(\frac{2\pi\omega_n}{\omega_d}\right)}}$$

$$= e^{\xi\left(-\omega_n t_1 + \omega_n t_1 + \frac{2\pi\omega_n}{\omega_d}\right)}$$

$$= e^{\xi\left(\frac{2\pi\omega_n}{\omega_d}\right)}$$ sub $\omega_d = \sqrt{1-\xi^2}\,\omega_n$

$$\frac{x_1}{x_2} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

Similarly for x_2 & x_3

$$\frac{x_2}{x_3} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

Which mean the ratio of successive amplitude remains the same

$$\frac{x_i}{x_{i+1}} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

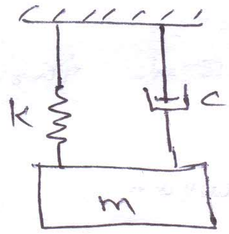
Taking logarithm

$$\log \frac{x_i}{x_{i+1}} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

This is known as logarithmic decrement

It is defined as the natural logarithm of the ratio of any two successive amplitudes on same side of the mean line.

Consider a spring mass system with viscous damping, while a harmonic force of frequency ω and amplitude F acts on it, as shown in fig



$\downarrow F \cos \omega t$ a harmonic force

ω → frequency of excitation

F → Amplitude of the force

The mass "m" is displaced from the equilibrium position

The equation of motion is

$$m\ddot{x} + c\dot{x} + kx = F \cos \omega t$$

$\div m$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F}{m} \cos \omega t$$

$\frac{c}{m} = 2\xi \omega_n$

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n x = \frac{F}{k}\frac{k}{m} \cos \omega t$$

$\frac{F}{k} = \frac{N}{N/m} = m$, so the disturbance is initial displacement

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n x = A\,\omega_n^2 \cos \omega_n t$$

A → Displacement Amplitude

$$x(t) = x_{c.s} + x_{P.I}$$

$x_{C.S}$ is similar or related to free damped motion, therefore it is going to die out in a course of time. Hence it is called the Transient Solution

$x_{P.I}$ is going to be Present throughout, therefore it is called Steady State Solution

The Solution take the form

phase angle

$$x(t) = X\cos(\omega t - \phi)$$

response amplitude

$$\dot{x}(t) = -X\omega\sin(\omega t - \phi)$$

$$\ddot{x}(t) = -X\omega^2\cos(\omega t - \phi)$$

Sub in the equation of motion

$$-X\omega^2\cos(\omega t - \phi) + 2\xi\omega_n\left[-X\omega\sin(\omega t - \phi)\right]$$

$$+\ \omega_n^2 X\cos(\omega t - \phi) = A\,\omega_n^2\cos\omega t$$

$$(-X\omega^2 + \omega_n^2 X)\cos(\omega t - \phi) - 2\xi\omega_n X\omega\left[\sin(\omega t - \phi)\right]$$

$$= A\,\omega_n^2\cos\omega t$$

$$(\cos\omega t\cos\phi + \sin\omega t\sin\phi)(-X\omega^2 + \omega_n^2 X) - 2\xi\omega_n X\omega$$

$$(\sin\omega t\cos\phi - \cos\omega t\sin\phi) = A\,\omega_n^2\cos\omega t$$

Put $\omega t = 0$ $\quad\cos(0) = 1 \quad \sin(0) = 0$

$$(-X\omega^2 + \omega_n^2 X)\cos\phi + 2\xi\omega_n X\omega\sin\phi = A\,\omega_n^2 \quad —①$$

Put $\omega t = 90°$

$\pm$

$$\sin\phi\,(-x\omega^2 + \omega_n^2 x) - \cos\phi\,(2\xi\omega_n x\omega) = 0 \quad -(2)$$

Squaring & adding (1) & (2)

$$\cos^2\phi\,(-x\omega^2 + \omega_n^2 x)^2 + \sin^2\phi\,(2\xi\omega_n x\omega)^2$$

$$+ \cancel{2\sin\phi\cos\phi\,(-x\omega^2 + \omega_n^2 x)(2\xi\omega_n x\omega)} +$$

$$\sin^2\phi\,(-x\omega^2 + \omega_n^2 x) + \cos^2\phi\,(2\xi\omega_n x\omega)^2 -$$

$$\cancel{2\sin\phi\cos\phi\,(2\xi\omega_n x\omega)(-x\omega^2 + x\omega_n^2)}$$

$$= A^2\omega^4$$

$$\cos^2\phi + \sin^2\phi\,(-x\omega^2 + \omega_n^2 x)^2 + \cos^2\phi + \sin^2\phi\,(2\xi\omega_n x\omega)^2$$

$$= A^2\omega^4$$

$$(-x\omega^2 + \omega_n^2 x)^2 + (2\xi\omega_n x\omega)^2 = A^2\omega_n^4$$

$$x^2\left[(-\omega^2 + \omega_n^2)^2 + (2\xi\omega_n\omega)^2\right] = A^2\omega_n^4$$

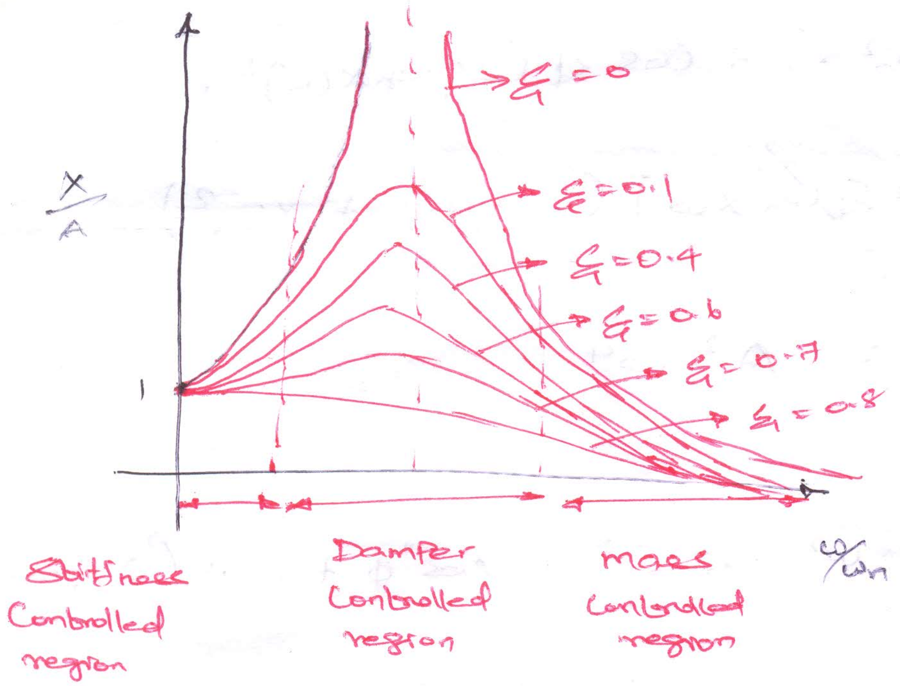
$$\frac{x^2}{A^2} = \frac{\omega_n^4}{(-\omega^2 + \omega_n^2)^2 + (2\xi\omega_n\omega)^2}$$

$$\frac{X^2}{A^2} = \frac{1}{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

$$\frac{X}{A} = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$X_p = A\frac{\cos(\omega t - \phi)}{\sqrt{(1-\omega^2/\omega_n^2)^2 + (2\xi\,\omega/\omega_n)^2}}$$

$\frac{X}{A}$ is called magnification factor



$$\omega = \omega_n\sqrt{1-2\xi^2}$$

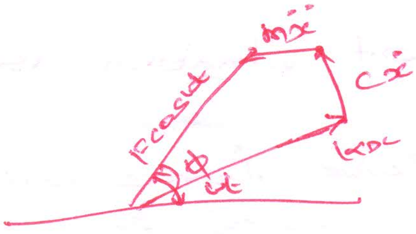
$$\sin\phi\,(-X\omega^2 + X\omega_n^2) - \cos\phi\,(2\xi\,\omega_n\omega X) = 0$$

$\div X$

$$\sin\phi\,(-\omega^2 + \omega_n^2) = \cos\phi\,(2\xi\omega\omega_n)$$

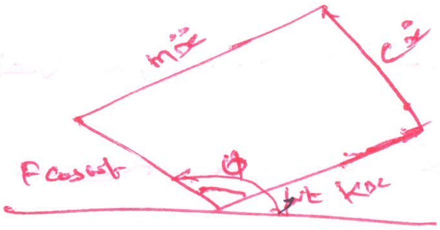
$$\tan\phi = \frac{2\xi\omega\omega_n}{-\omega^2+\omega_n^2} = \frac{2\xi(\omega/\omega_n)}{1-\omega^2/\omega_n^2}$$

(a) When $\omega/\omega_n << 1$



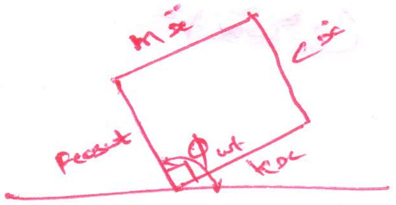
(c) $\phi \simeq 180°$

$\omega/\omega_n >> 1$



$\omega/\omega_n = 1$

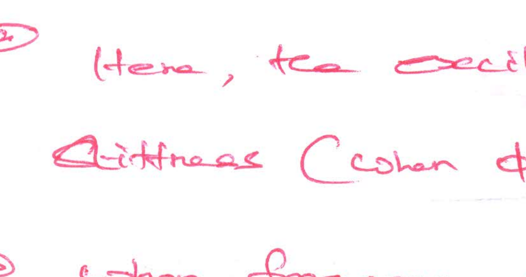
(b) $\phi \simeq 90°$

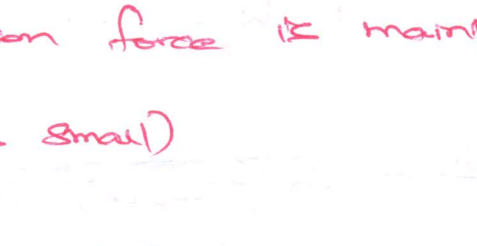


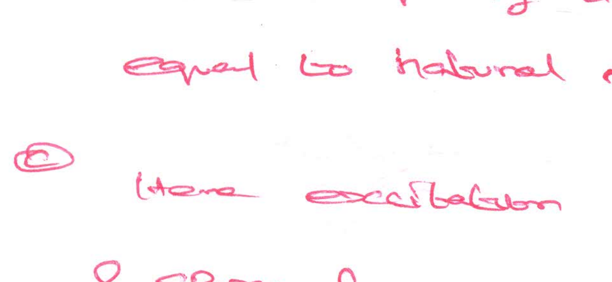
(a) Here, the excitation force is mainly balanced by stiffness (when ϕ is small)

(b) when frequency of excitation ω increases & becomes equal to natural frequency ω_n (when $\phi = 90°$)

(c) Here excitation force ω, very high than Damping & spring force are small in magnitude







TRANSMISSIBILITY

The amplitude of vibration is X, so the maximum value of those force will be kX & $c\omega X$ respectively.

$$F_T = \sqrt{(kx)^2 + (c\omega x)^2}$$

$$= x\sqrt{k^2 + c^2\omega^2}$$

$$= kx\sqrt{1 + \frac{c^2\omega^2}{k^2}}$$

$$\frac{c^2\omega^2}{k^2} = \frac{(2\xi\omega_n m)^2\omega^2}{k^2} \qquad c = 2\xi\omega_n m$$

$$= \frac{4\xi^2\omega_n^2\omega^2}{\omega_n^4} = \frac{4\xi^2\omega^2}{\omega_n^2}$$

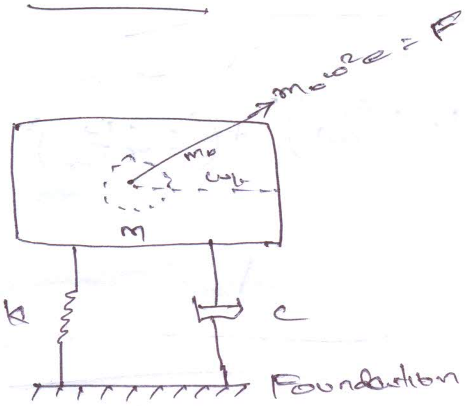
$$F_T = kx\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}$$

$$\frac{X}{A} = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\,\omega/\omega_n\right)^2}} \Rightarrow \frac{A}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\xi\,\omega/\omega_n\right)^2}}$$

Sub $\cancel{x}$ in F_T

$$F_T = \frac{kA\sqrt{1 + (2\xi\,\omega/\omega_n)^2}}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\xi\,\omega/\omega_n\right)^2}} \Rightarrow \frac{F_T}{F} = \frac{\sqrt{1 + (2\xi\,\omega/\omega_n)^2}}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\xi\,\omega/\omega_n\right)^2}}$$

Rotating unbalance



A machine having rotor as one of its components is called a rotating machine.

When centre of gravity of rotor does not coincide with the axis of rotation, the problem occurs unbalanced.

The distance b/w axis of rotation & the centre of gravity is called eccentricity e and the mass acting at a distance e from the axis of rotation is known as eccentric mass m_0.

$$m\ddot{x} + c\dot{x} + kx = F \sin\omega t$$

$$\ddot{x} + \frac{c}{m} + \frac{k}{m}x = \frac{m_0}{m} e\omega^2 \sin\omega t$$

$$= \frac{m_0}{m} e \frac{\omega^2}{\omega_n^2} \omega_n^2 \sin\omega t$$

$$= A\,\omega_n^2 \sin\omega t$$

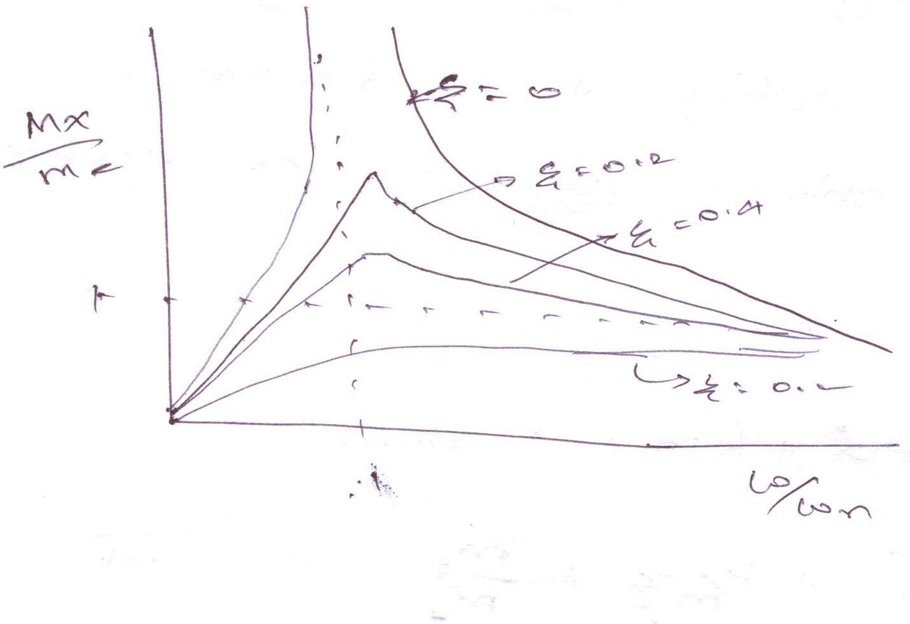
$$\frac{X}{A} = \frac{1}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\frac{X}{\frac{m_o}{M} e \frac{\omega^2}{\omega_n^2}} = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

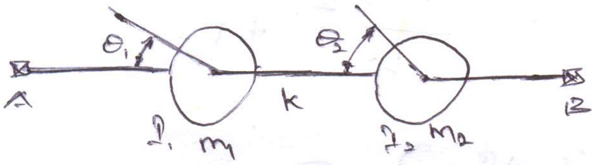
$$\frac{M X}{m_o e} = \frac{\omega^2/\omega_n^2}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

At resonance $\omega = \omega_n$

$$\frac{MX}{m_o e} = \frac{1}{2\xi} \Rightarrow X = \frac{m_o e}{2M\xi}$$



Two degree of freedom Torsional vibration.



Consider a shaft AB is carrying two rotors having moment of inertia I_1 & I_2

θ_1 & θ_2 be the angular displacement of the rotors

$$I_1 \frac{d^2\theta_1}{dt^2} + k(\theta_1 - \theta_2) = 0$$

$$I_2 \frac{d^2\theta_2}{dt^2} + k(\theta_2 - \theta_1) = 0$$

Assuming

$$\theta_1 = A_1 \sin \omega t \qquad \theta_2 = A_2 \sin \omega t$$

$$\ddot{\theta}_1 = -\omega^2 A_1 \sin \omega t \qquad \ddot{\theta}_2 = -\omega^2 A_2 \sin \omega t$$

$$(-\omega^2 I_1 + k)a_1 - k a_2 = 0$$

$$(-\omega^2 I_2 + k)a_2 - k a_1 = 0$$

$$(-\omega^2 I_1 + k)(-\omega^2 I_2 + k) - k^2 = 0$$

$$\omega^2(\omega^2 I_1 I_2 - I_1 k - I_2 k) = 0$$

$$\omega^2\left[\omega^2 - \frac{k}{I_1 I_2}(I_1 + I_2)\right] = 0$$

$\omega_1 = 0 \qquad \omega_2 = \sqrt{\dfrac{K(I_1+I_2)}{I_1 I_2}}$

$$\frac{a_1}{a_2} = \frac{-\omega^2 I_2 + K}{K} = -\frac{\omega^2 I_2}{K} + 1$$

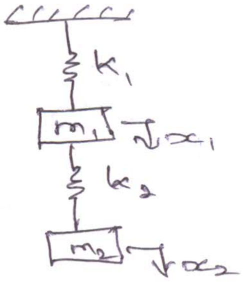
$$= -\frac{K(I_1+I_2)}{I_1 I_2} \cdot \frac{I_2}{K} + 1$$

$$\frac{a_1}{a_2} = -\frac{I_2}{I_1}$$

SEMI - DEFINITE

LAGRANGE'S EQUATIONS

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_j}\right) - \frac{\partial T}{\partial x_j} + \frac{\partial U}{\partial x_j} = Q_j$$



T - total kinetic energy

V - total Potential energy

$j = 1, 2, 3 \ldots n$

n - degree of freedom

Q_j - external force.

Kinetic & Potential energy of the system.

$$T = \tfrac{1}{2} m_1 \dot{x}_1^2 + \tfrac{1}{2} m_2 \dot{x}_2^2$$

$$V = \tfrac{1}{2} k_1 x_1^2 + \tfrac{1}{2} k_2 (x_2 - x_1)^2$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$$

$$\frac{\partial T}{\partial x_1} = 0$$

$$\frac{\partial V}{\partial x_1} = k_1 x_1 - k_2 (x_2 - x_1)$$

First equation of motion

$$m_1 \ddot{x}_1 + k_1 x_1 - k_2 (x_2 - x_1) = 0 \longrightarrow (1)$$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) = 0$$

For second equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \; ; \; \frac{\partial T}{\partial x_2} = 0 \; ; \; \frac{\partial V}{\partial x_2} = k_2 (x_2 - x_1)$$

Amplitude ratio

$$\frac{A_1}{A_2} = \frac{k}{k+k_1-m_1\omega^2} = \frac{k}{2k-m\omega^2}$$

&

$$\frac{A_1}{A_2} = \frac{k_2+k-m_2\omega^2}{k} = \frac{2k-m\omega^2}{k}$$

$\omega = \sqrt{k/m}$

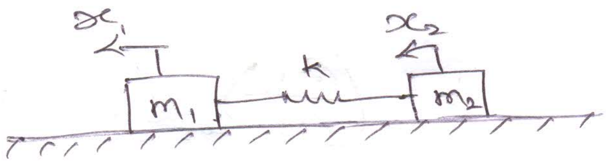
$$\Rightarrow \frac{A_1}{A_2} = 1$$

$\omega = \sqrt{3k/m}$

$$\Rightarrow \frac{A_1}{A_2} = -1$$

SEMI-DEFINITE SYSTEM

The systems having one of their natural frequencies equal to zero are known as semi-definite system.



$$m_1 \ddot{x}_1 + k(x_1 - x_2) = 0$$

$$m_2 \ddot{x}_2 + k(x_2 - x_1) = 0$$

$$x_1 = A_1 \sin(\omega t + \phi) \qquad x_2 = A_2 \sin(\omega t + \phi)$$

$$\ddot{x}_1 = -\omega^2 A_1 \sin(\omega t + \phi) \qquad \ddot{x}_2 = -\omega^2 A_2 \sin(\omega t + \phi)$$

$$(k - m_1\omega^2) A_1 - k A_2 = 0$$

$$-k A_2 + (k - m_2\omega^2) A_2 = 0$$

$$\begin{vmatrix} k - m_1\omega^2 & -k \\ -k & k - m_2\omega^2 \end{vmatrix} = 0$$

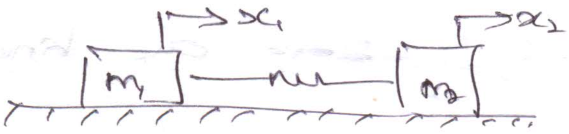
$$m_1 m_2 \omega^4 - k(m_1 + m_2)\omega^2 = 0$$

$$\omega^2 \left[m_1 m_2 \omega^2 - k(m_1 + m_2) \right] = 0$$

$$\omega_1 = 0 \quad ; \quad \omega_2 = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$$

(1) Solve the Problem shown in fig 5.22 $m_1 = 60$ kg

$m_2 = 15$ kg & $k = 320$ N/m

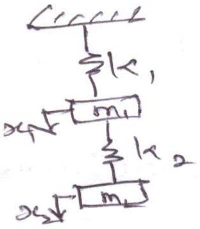


Ans :-

$\omega_1 = 0$; $\omega_2 = 7.80$ rad/sec

$\left(\frac{A_1}{A_2}\right)_{\omega_1} = 1.0$; $\left(\frac{A_1}{A_2}\right)_{\omega_2} = -1.49$

(2)



$m_1 = 1.5$ kg

$m_2 = 0.80$ kg

$k_1 = k_2 = 40$ N/m

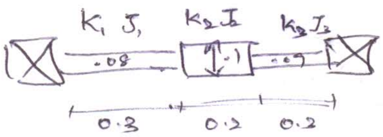
Ans!

$\omega^4 - 103.33\,\omega^2 + 1333.33 = 0$

$\omega_1 = 9.39$ rad/sec ; $\omega_2 = 3.88$ rad/sec

$\left(\frac{A_1}{A_2}\right)_{\omega_1} = -0.765$ $\left(\frac{A_1}{A_2}\right)_{\omega_2} = 0.696$

(3) Two bodies having equal masses as 60 kg each and radius of gyration 0.3 m are keyed to both ends of a shaft .80 m long. The shaft is 0.08 m in diameter for 0.3 m length, 0.10 m diameter for 0.2 m length and .9 m dia for rest of the length. Find the frequency of torsional vibration $G = 9 \times 10^{11}$ N/m²



$I = mk^2 = 60 \times .3 \times .3 = 5.4$ kg m²

$k_1 = \frac{G J_1}{l_1} = \frac{9 \times 10^{11} \times \pi/32\, d^4}{.3} = 1.20 \times 10^7$ N-m/rad

$k_2 = 4.415 \times 10^7$ N-m/rad $k_3 = 1.931 \times 10^7$ N-m/rad

$k = 6.356 \times 10^6$ N m/rad $\omega = \sqrt{\frac{k(I_1+I_2)}{I_1 I_2}} = 1.534 \times 10^3$ rad/s

$$a^2k^2 + (-k + \omega^2 m)(-\omega^2 I + a^2k + k_T) = 0$$

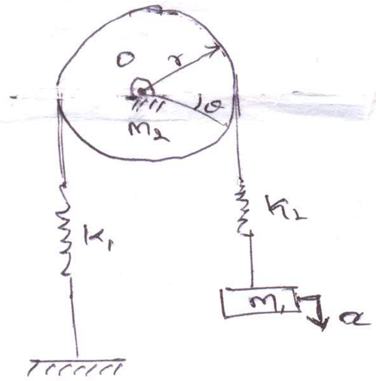
$$\omega^4 - \omega^2\left(k/m + \frac{a^2k}{I} + \frac{k_T}{I}\right) + \frac{k\,k_T}{Im} = 0$$

$$\omega^4 - 4749.99\omega^2 + 3333333.3 = 0$$

$$\omega_1 = 62.40 \text{ rad/sec}; \quad \omega_2 = 29.25 \text{ rad/sec}$$

Find the natural frequencies of the system shown in fig. Assume that there is no slip between the cord and cylinder

$k_1 = 40$ N/m ; $k_2 = 60$ N/m ; $m_1 = 2$kg ; $m_2 = 10$kg



$$m_1\ddot{x} + k_2(x - r\theta) = 0$$

$$I\ddot{\theta} - k_2(x - r\theta)r + k_1 r^2\theta = 0$$

$$m_1\ddot{x} + k_2 x - k_2 r\theta = 0$$

$$I\ddot{\theta} + (k_1 r^2 - k_2 r^2)\theta - k_2 x r = 0$$

$$x = A\sin\omega t \quad ; \quad \ddot{x} = -\omega^2 A\sin\omega t$$

$$\theta = \phi\sin\omega t \quad ; \quad \ddot{\theta} = -\omega^2\phi\sin\omega t$$

$$(k_2 - \omega^2 m_1)A - k_2 r\phi = 0$$

$$(k_1 r^2 + k_2 r^2 - \omega^2 I)\phi - k_2 r A = 0$$

$$I = \tfrac{1}{2} m_2 r^2$$

$$\omega^4 - \omega^2\left[\frac{2(k_1 + k_2)}{m_2} + \frac{k_2}{m_1}\right) + \frac{2k_1k_2}{m_1m_2} = 0$$

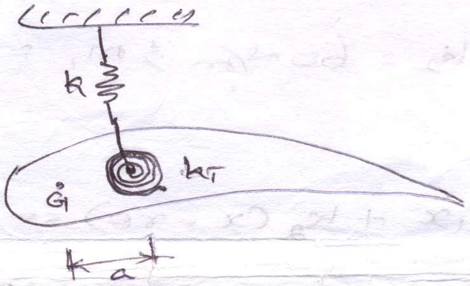
$$\omega^4 - 50\omega^2 + 240 = 0$$

$$\omega_1 = 6.68 \text{ rad/sec} \quad \omega_2 = 2.32 \text{ rad/sec}$$

An aerofoil wing in its first bending and torsional modes can be represented schematically as shown in fig connected through a translational spring of stiffness k and a torsional spring of stiffness k_T. Write the equations of motion for the system and obtain the two natural frequencies. Assume the following data

$m = 5\,\text{kg}$; $I = 0.12\ \text{kg m}^2$; $k = 50\times10^3\ \text{N/m}$,

$k_T = 0.4\times10^2\ \text{N m/rad}$ $\quad a = 0.1\ \text{m}$



$$m\ddot{x} = -k(x + a\theta)$$

$$m\ddot{x} + k(x + a\theta) = 0$$

$$I\ddot{\theta} = -ak(x + a\theta) - k_T\theta$$

$$I\ddot{\theta} + ak(x + a\theta) + k_T\theta = 0$$

$x = A\sin\omega t$; $\quad \ddot{x} = -\omega^2 A\sin\omega t$

$\theta = \phi\sin\omega t$ $\quad \ddot{\theta} = -\omega^2\phi\sin\omega t$

$$-\omega^2 mA + k(A + a\phi) = 0$$

$$\frac{A}{\phi} = \frac{ak}{(-k + \omega^2 m)}$$

$$-\omega^2 I\phi + ak(A + a\phi) + k_T\phi = 0$$

$$\frac{A}{\phi} = \frac{-\omega^2 I + a^2k + k_T}{-ak}$$

Vibration isolation

The Machine when mounted on foundation and supports cause vibrations because of unbalanced forces, which damage the foundation on which the machines are mounted. So the vibrations transmitted to the foundation should be eliminated or reduced using some device such as spring, damper etc.

The two basic requirement for an isolator.

1. There should be no rigid connection b/w the unit and the base.

2. It should be ensured that the isolator remains together in case the damping material fails.

Isolation materials

Rubber, felt, cork, metallic spring etc.

Rubber:

useful for shear loads

Transmissibility is low

It used in light loads and high frequency oscillations

It cannot be in high temperature, gasoline & oil because of its properties changes

Felt

Damping factor is high

used in low frequency ratio

uses many pad's instead of single large pad.

Cork:

It suitable for compressive loads.

At high load it becomes more flexible

Metal spring:

Helical spring & leaf spring

It has high sound transmissibity

It suitable for all condition because

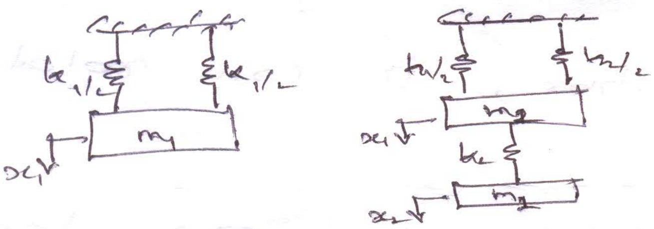
it not affected by air, water, oil or temperature

useful for high frequency ratio.

Vibration Absorber

When a structure externally excited has undesirable vibrations, it becomes necessary to eliminate them by coupling some vibrating system to it. The vibrating system is known as vibration absorber.

Dynamic vibration absorber



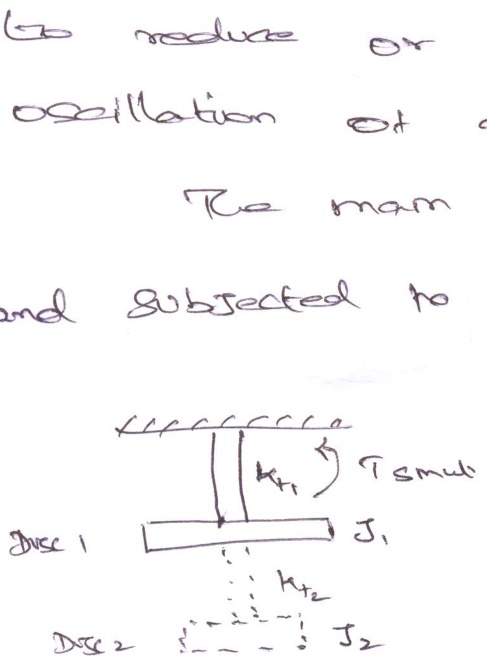
The spring mass system k_2, m_2 is coupled to the main system.

$$\omega = \sqrt{\frac{k_2}{m_2}} = \omega_2$$

Torsional vibrational Absorber

Torsional vibrational absorber can be used to reduce or completely eliminate torsional oscillation of a system

The main system represented by k_1 & J_1 and subjected to periodic torque $T \sin \omega t$



k_{t_2} & J_2 for torsional vibration absorber

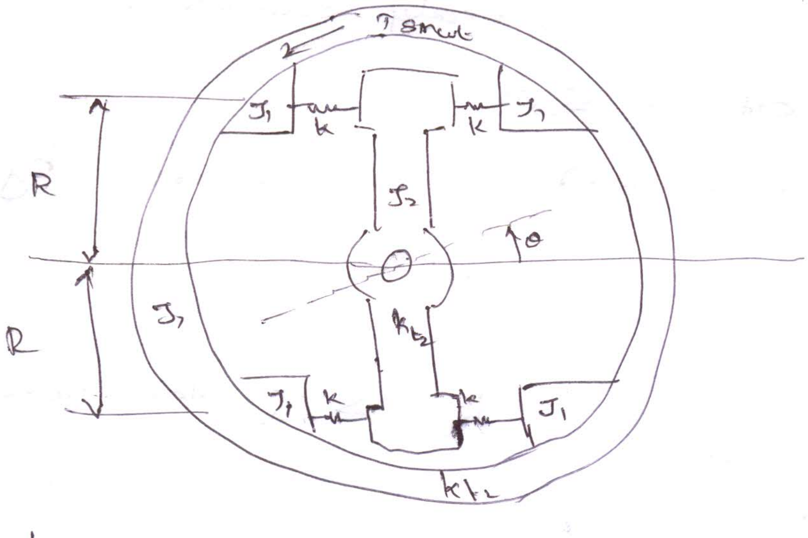
Due to limited space, J_2 can ~~be~~ not be implemented but shaft with stiffness k_{t_2} can added to J_1 to absorb the impressed torque.

But shaft with too much length cannot be possible.

<u>Ring Torsional absorber</u>

It consists of a ring attached to one of the disc of the original system. The mass is connected to the ring by means of spring. If no vibration, it will rotates at constant speed.

If there is vibration, the spring tends to deflected and act as a absorber, and that causes change in their Potential energy and absorbs the energy of main system.



$$k_{t_2} = 4 \times k \times R^2$$

$$k_e = k + k + k + k = 4k$$

Spring force

$$F = k_e \times x = 4k \times x$$

$$= 4k\,R\theta \qquad x = R\theta$$

Torque exerted on the absorber

$$T_2 = F \times R = 4k\,(R\theta)\,(R)$$

$$= 4k\,R^2\theta$$

$$T = \beta_2\,k_{Ec}$$

T - maximum value of applied torque

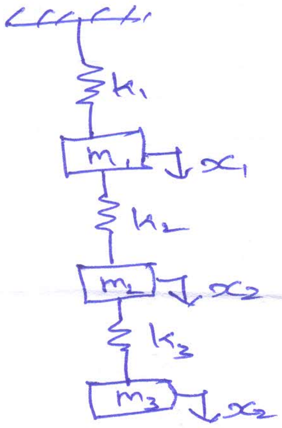
B_2 = amplitude of vibration

Multi degrees of freedom

The system having more than one degree of freedom are known as several or multi degree of freedom.

The system must also have many equation of motion and as many natural frequencies.

MATRIX METHOD



$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = 0$$

$$m_2\ddot{x}_2 - k_2x_1 + (k_3+k_2)x_2 - k_3x_3 = 0$$

$$m_3\ddot{x}_3 - k_3x_2 + k_3x_3 = 0$$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 & 0 \\ -k_2 & k_3+k_2 & -k_3 \\ 0 & -k_3 & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 0$$

$$[m]\{\ddot{x}\} + [k]\{x\} = 0$$

$[m]$ = mass matrix $\quad$ $[k]$ = Stiffness matrix

$$\{\ddot{x}\} + [m]^{-1}[k]\{x\} = 0$$

$$\{\ddot{x}\} + [C]\{x\} = 0$$

$[C] = [m]^{-1}[k]$ = Dynamic matrix & $[m]^{-1} = \dfrac{\text{adj } m}{|M|}$

For harmonic oscillation at frequency ω,

$$\{\ddot{x}\} = -\omega^2\{x\}$$

So,

$$[C]\{x\} - \omega^2\{x\} = 0$$

If $\omega^2 = \lambda$, the equation

$$[C]\{x\} - \lambda\{x\} = 0$$

$$[\lambda I - C]\{x\} = 0$$

I = identity matrix

Solution of equation may obtain by

$$|\{\lambda I - C\}| = 0$$

Now

$m_1 = m_2 = m_3 = m$ & $k_1 = k_2 = k_3 = k$

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 2k & -k & 0 \\ -k & -2k & -k \\ 0 & -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 0$$

$$|m| = m^3$$

$$m^{-1} = \frac{\text{adj } m}{|m|} = \frac{1}{m^3} \begin{bmatrix} m^2 & 0 & 0 \\ 0 & m^2 & 0 \\ 0 & 0 & m^2 \end{bmatrix}$$

$$C = [m^{-1}][k]$$

$$= \frac{1}{m}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}$$

$$[C] = \frac{k}{m}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$|\lambda I - C| = 0$$

$$\begin{vmatrix} \lambda - 2k/m & k/m & 0 \\ k/m & \lambda - 2k/m & k/m \\ 0 & k/m & \lambda - k/m \end{vmatrix} = 0$$

$$\lambda^3 - 5\lambda^2 k/m + 6\lambda\, k^2/m^2 - k^3/m^3 = 0$$

$$\lambda_1 = 0.198\, k/m = \omega_1^2$$

$$\omega_2 = \sqrt{1.555\, k/m} \quad ; \quad \omega_3 = \sqrt{3.247\, k/m}$$

Continuous System

There are some system such as beams cables, rods etc, which have their mass & elasticity distributed continuously throughout the length. Such systems are known as continous system.

This system which has infinite number of particles hence it has infinite number of degree of freedom.

Boundary Conditions

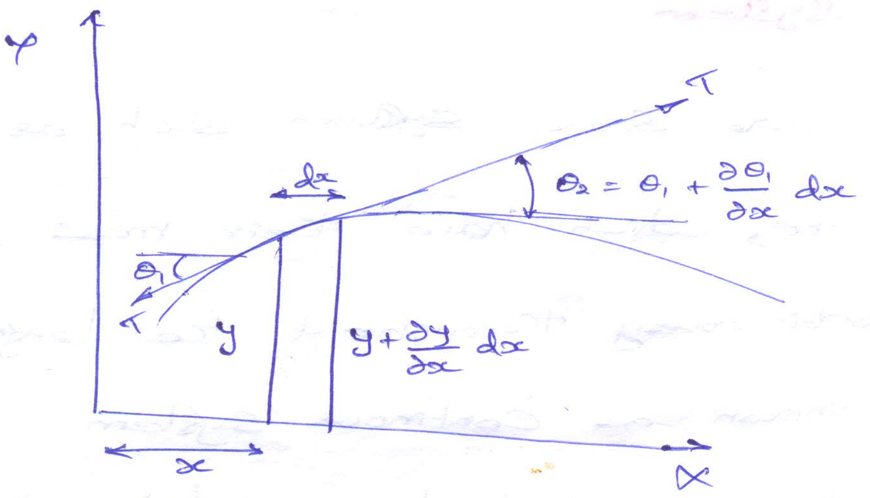
The system which has uniform cross section having homogeneous & isotropic material.

Boundary conditions may be geometric or natural or both.
(Bar fixed both ends)
(force & moments)

Lateral Vibration of a String

Consider a vibrating string of mass ρ per unit length having transverse vibration under tension T as shown in fig

It assumed that for a very small amplitude of string vibration the tension T remains constant throughout.



$$\theta_1 = \frac{\partial y}{\partial x} \qquad \theta_2 = \frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) dx$$

$$\theta_2 = \theta_1 + \frac{\partial \theta_1}{\partial x} dx$$

Resolving the tension along y axis

$$T \sin\left(\theta_1 + \frac{\partial \theta_1}{\partial x} dx\right) - T \sin\theta_1 = \text{mass} \times \text{accelerat}$$

$$T\left(\theta_1 + \frac{\partial \theta_1}{\partial x} dx\right) - T\theta_1 = \rho \, dx \frac{\partial^2 y}{\partial t^2}$$

$$T \frac{\partial \theta_1}{\partial x} dx = \rho \, dx \frac{\partial^2 y}{\partial t^2}$$

$$T \frac{\partial \theta_1}{\partial x} = \rho \frac{\partial^2 y}{\partial t^2}$$

$$\frac{T}{\rho} \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) = \frac{\partial^2 y}{\partial t^2}$$

$$\frac{T}{\rho} \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t}$$

Assume $\frac{T}{\rho} = a^2$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{a^2} \frac{\partial^2 y}{\partial t^2}}$$

This is one-dimensional wave equation for lateral vibrations of string.

The lateral deflection Y along the string is a function of variables x & t. so it can be written as

$$y = y(x,t)$$

$$y(x,t) = X(x)\,T(t)$$

Substituting in one dimensional wave equation

$$\frac{a^2}{X} \cdot \frac{d^2X}{dx^2} = \frac{1}{T}\frac{d^2T}{dt^2}$$

L.H.S function of x } equating to some
R.H.S function of t } constant $-p^2$

$$\frac{d^2X}{dx^2} + \left(\frac{p}{a}\right)^2 X = 0 \quad \&$$

$m^2+1=0$

$m = \pm 1$

$$\frac{d^2T}{dt^2} + Tp^2 = 0$$

$$X(x) = A\cos\left(\frac{p}{a}\right)x + B\sin\left(\frac{p}{a}\right)x$$

$$T(t) = C\cos pt + D\sin pt$$

The general solution can be written as

$$\boxed{Y(x,t) = \sum_{n=1}^{\infty}\left[A_n\cos\left(\frac{p}{a}\right)x + B_n\sin\left(\frac{p}{a}\right)x\right]\left[C_n\cos pt + D_n\sin pt\right]}$$

$y(x,t) = X(x)\,T(t)$

$T = \text{Const} \qquad X = \text{variable}$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) = \frac{\partial}{\partial x} T\left(\frac{\partial X}{\partial x}\right)$$

$$\frac{\partial^2 y}{\partial x^2} = T\,\frac{\partial^2 X}{\partial x^2}$$

$$\frac{d^2 y}{dx^2} = T\,\frac{d^2 X}{dx^2}$$

$X = \text{const} \qquad T = \text{variable}$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial y}{\partial t}\right)$$

$$= \frac{\partial}{\partial t} X\left(\frac{\partial T}{\partial t}\right)$$

$$\frac{\partial^2 y}{\partial t^2} = X\,\frac{\partial^2 T}{\partial t^2}$$

$$\frac{d^2 y}{dt^2} = X\,\frac{d^2 y}{dt^2}$$

In this equation P is the ~~fre~~ frequency of vibration. A_n, B_n, C_n & D_n are the arbitrary Parameter

A vibrating system is defined by the following parameters $m = 3\,kg$, $k = 100\,N/m$, $C = 3\,Ns/m$. Determine (a) the damping factor (b) the natural frequency of damped vibration (c) logarithmic decrement (d) the ratio of two consecutive amplitude and (e) the number of cycles after which the original amplitude is reduced to 20 percent

$$C_c = 2\sqrt{km} = 2\sqrt{100 \times 3} = 34.64\ Ns/m$$

(a) $$\varepsilon = \frac{C}{C_c} = \frac{3}{34.64} = 0.086$$

(b) $$\omega_d = \omega\sqrt{1-\varepsilon^2} = 5.75\ rad/sec$$

$$f_d = \frac{\omega_d}{2\pi} = 0.92\ Hz$$

(c) $$\delta = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}} = 0.542$$

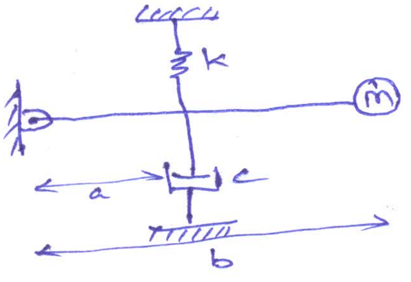
(d) $$\delta = \ln\frac{x_1}{x_2} \Rightarrow \frac{x_1}{x_2} = e^{\delta} = 1.72$$

(e) $$\delta = \frac{1}{n}\ln\frac{x_1}{x_n}$$

$$n = \frac{1}{\delta}\ln\frac{x_1}{x_n} = \frac{1}{0.542}\ln\left(\frac{x_1}{x_1/5}\right) = \frac{1}{0.542}\ln 5$$

$$n = 2.96\ \text{cycles}$$

Derive equation of motion for the system shown in figure. If $m = 1.5\,kg$, $k = 4900\,N/m$, $a = 6\,cm$ & $b = 14\,cm$ determine the value of c_c for which the system is critically damped.



Solution :

The displacement of spring $= a\theta = x$

Spring force $= kx = ka\theta$

damping force $= c\dot{x} = ca\dot{\theta}$

mass $= m\ddot{x} = mb\ddot{\theta}$

$$I\ddot{\theta} + (ka\theta)a + (ca\dot{\theta})a = 0 \qquad Ib\ddot{\theta} + ka\theta + ca\dot{\theta} = 0$$

$$mb^2\ddot{\theta} + ca^2\dot{\theta} + ka^2\theta = 0$$

$$\ddot{\theta} + \frac{ca^2\dot{\theta}}{mb^2} + \frac{ka^2}{mb^2}\theta = 0$$

$$= -\frac{ca^2}{2mb^2} \pm \sqrt{\left(\frac{ca^2}{2mb^2}\right)^2 - \frac{ka^2}{mb^2}}$$

$$\frac{ca^2}{2mb^2} = \frac{a}{b}\sqrt{\frac{k}{m}}$$

$$C_c = \frac{2b}{a}\sqrt{km} \Rightarrow \frac{2 \times 14}{b}\sqrt{4900 \times 1.5}$$

$$C_c = 400\ Ns/m$$

A horizontal spring mass system with Coulomb damping has a mass of 5 kg attached to a spring of stiffness 980 N/m. If the coefficient of friction is 0.025 calculate (a) the frequency of free oscillations (b) the no of cycles corresponding to 50% reduction in amplitude, if the initial amplitude is 5.0cm (c) the time taken to achieve this 50% reduction.

$F = \mu mg = 0.25 \times 5 \times 9.81 = 1.226N$

(a) $\omega_n = \sqrt{\frac{980}{5}} = 14 rad/sec \quad f_n = 2.23\ Hz$

(b) Reduction in amplitude/cycle $= \frac{4F}{k}$

$$\frac{4 \times 1.226}{980} = 5 \times 10^{-3} m$$

Cycle completed in 50% $= \frac{M}{5 \times 10^{-3} m} = 5 cycles$

(c) Time taken to achieve 50%

$= \text{No of cycle} \cdot \frac{2\pi}{\omega_n}$

$= 5 \times \frac{2\pi}{14} = 2.24 sec$

VIBRATION ISOLATION

The Engine and Machines when mounted on foundation and supports causes vibrations of excessive amplitude becauses of unbalanced forces.

This disturbing forces which damage the foundation on which the machines are mounted.

So the vibration should be eliminated or reduced to Prevent damage

Requirement for Isolator:

No rigid connection between the unit and base

It should be remain together and should keep the unit in safe Position when damping fails.

The material used are:

Rubber, felt cork, metallic spring etc.

Rubber as an Isolator is quite useful for shear loading. It cannot be used in high temperature in the Presence of gasoline and oil.

It Prefered for light load and high oscillations

Felt: The damping factor is high, specially used for low frequency. Use of many pads at a time.

Cork: It is suitable for compressive load. At high load it becomes more flexible

Metal Spring:

Helical spring & leaf spring

It has high sound transmissibity which can cover by rubber or felt. Used for high frequency ratio

VIBRATION ABSORBER

When a structure externally ~~excited~~ excitation is high, it becomes necessary to eliminate them by coupling some vibrating system.

The vibration ~~absorber~~ system is called vibration absorber, K.E @ mean position $= \frac{1}{2} m \dot{x}^2 = \frac{1}{2} m \omega_n^2 A^2$.

Max. P.E @ extreme position $= \frac{1}{2} k A^2$,

$$\frac{1}{2} m \omega_n^2 A^2 = \frac{1}{2} K A^2.$$

$$\omega^2 = k/m.$$

$$\omega = \sqrt{k/m} \quad (\text{rad/sec}).$$

Addition of Harmonic Motion:

Add two harmonic motions of the same frequency, we get the resultant motion as harmonic.

$$x_1 = A_1 \sin \omega t \quad , \quad x_2 = A_2 \sin(\omega t + \phi)$$

having, same amplitude-A, same frequency-ω & phase diff.-ϕ.

Resultant motion, $x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$

$$x = A_1 \sin \omega t + A_2 \sin \omega t \cos\phi + A_2 \cos \omega t \sin \phi.$$

$$= \sin \omega t (A_1 + A_2 \cos \phi) + A_2 \cos \omega t \sin \phi.$$

Assume, $A_1 + A_2 \cos\phi = A \cos\theta$ —(1) & $A_2 \sin \phi = A \sin\theta$ —(2)

$$\therefore x = A \sin \omega t \cos\theta + A \cos \omega t \sin\theta.$$

$$= A (\sin \omega t + \theta).$$

Find A $\Rightarrow$ Squaring eqn (1) & adding eqn (2) $\Rightarrow$ $A^2 = (A_1 + A_2 \cos\phi)^2 + A_2^2 \sin^2\phi$

$$= A_1^2 + A_2^2 \cos^2\phi + 2A_1 A_2 \cos\phi + A_2^2 \sin^2\phi$$

$$A = (A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi)^{1/2}$$

Find θ $\Rightarrow$ (2) $\div$ (1) $\Rightarrow$ $\frac{A \sin\theta}{A \cos\theta} = \frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$

$$\tan\theta = \frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}\right).$$

Newton's Method:

Si

D'Alembert's Principle: States that the resultant force acting along with the inertial force is zero, then the body will be in static equilibrium.

Inertial force acting on the body, $F_i = m.f$.

m - mass of the body.

f - linear acceleration of the centre of mass.

The body will be in static equilibrium, if.

$$F + F_i = 0.$$

Spring - mass system in Horizontal position:

According to Newton's II law.

mass x acceleration = resultant force on the mass.

$$m\ddot{x} = -kx.$$

Rayleigh's Method:

It is assumed that the max. K.E at mean position is equal to the max. P.E at the extreme position.

The motion is assumed to be simple harmonic.

$$x = A \sin \omega_n t.$$

x - displacement of the body from mean position after time t.

A - max. displacement from mean position to the extreme position.

$$\dot{x} = \omega_n A . \cos \omega_n t.$$

Max. velocity $\left.\begin{matrix}\\ \text{@ mean position}\end{matrix}\right\}$ $\dot{x} = \omega_n A.$

Value of Static deflection (δ) for various types of Beams and under various load condition

W - load, l - length of the shaft or beam

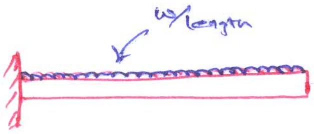
E - Young's modulus for the material of the shaft N/m^2

I - moment of inertia of the shaft $\left(I = \frac{\pi}{64} d^4\right)$

1.

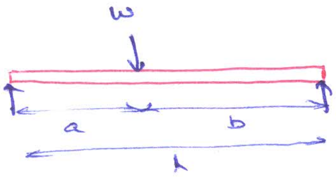
$$\delta = \frac{W l^3}{3EI}$$ (at free end)

2.



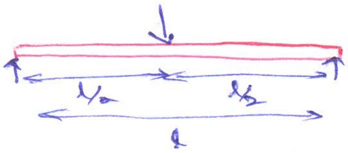
$$\delta = \frac{w l^4}{8EI}$$ (at the free end)

3.



$$\delta = \frac{W a^2 b^2}{3EI}$$ (Point load)

4.

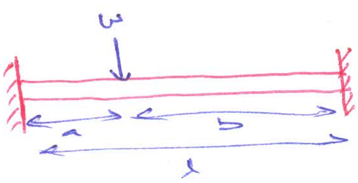


$$\delta = \frac{W l^3}{48EI}$$ (at centre)

5.

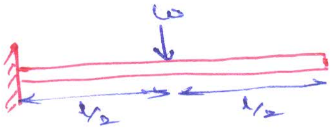
$$\delta = \frac{5}{384} \times \frac{w l^4}{EI}$$ (at centre)

6.



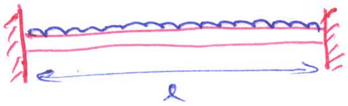
$$\delta = \frac{W a^3 b^3}{3EI l^3}$$

7.



$$\delta = \frac{W l^3}{192 EI}$$

8.



$$\delta = \frac{w l^4}{384 EI}$$

Problem:

1. A cantilever shaft 50 mm diameter and 300 mm long has a disc of mass 100 kg at its free end. The Young's modulus for the shaft material is 200 GN/m². Determine the frequency of the shaft (Ans f_n = 41 Hz)

Free transverse vibration due to a point load acting over a beam

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

2. A shaft of length 0.75 m supported freely at the end, is carrying a body of mass 90 kg at 0.25 m from one end. Find the natural frequency of transverse vibration (Assume E = 200 GN/m² and diameter = 50 mm)

(Ans :- 49.85 Hz)